Carleton University
Canada's Capital University

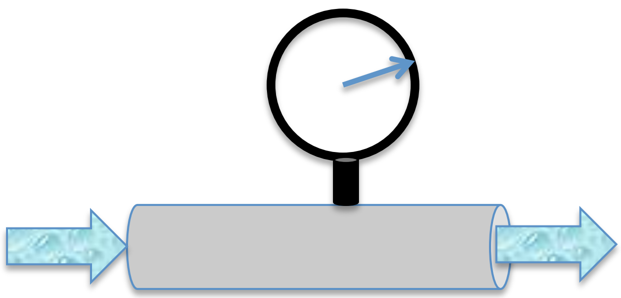
## Mathematics Tutorial Series

Numerical Integration - 1

First, "integration" here means something like

"Arriving at a whole by building up from the parts"

Suppose we want the total flow of water in a pipe and have only a gauge that measures rate of flow (in $m^3/sec$ say).



Suppose we measure:

| Time | Flow rate $f(t)$ at time $t$ |
| --- | --- |
| $t = 0$ | $7\ m^3/sec$ |
| $t = 5$ | $10\ m^3/sec$ |
| $t = 10$ | $4\ m^3/sec$ |

What is the total flow from $t = 0$ to $t = 10$?
We write the exact flow as:

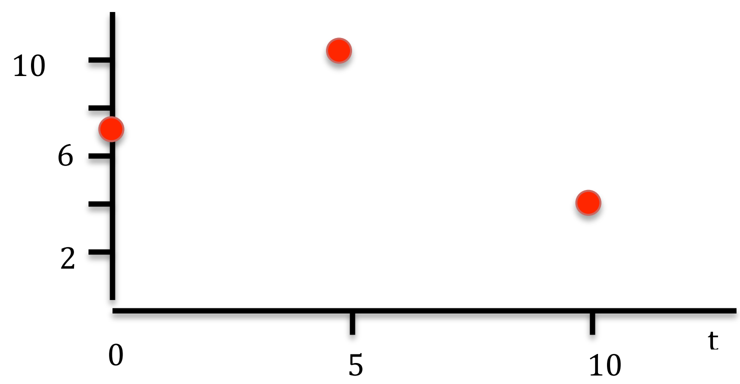
$$\int_{t=0}^{t=10} f(t)\, dt$$

From the measurements we can only approximate.

Two issues:
1. How shall we approximate?
2. How shall we make the approximation better?

**Approximation**

| Time | Flow rate $f(t)$ at time $t$ |
|---|---|
| $t = 0$ | $7\ m^3/sec$ |
| $t = 5$ | $10\ m^3/sec$ |
| $t = 10$ | $4\ m^3/sec$ |




Scenario 1. Use the average flow rate from $t = 0$ to $t = 10$.

$$10\,\frac{7 + 10 + 4}{3} = 70\ m^3$$

Scenario 2. Use the average flow on each of the two intervals

First interval, average flow rate is $\frac{7+10}{2} = 8.5\ m^3/sec$

Second interval average flow is $\frac{10+4}{2} = 7\ m^3/sec$

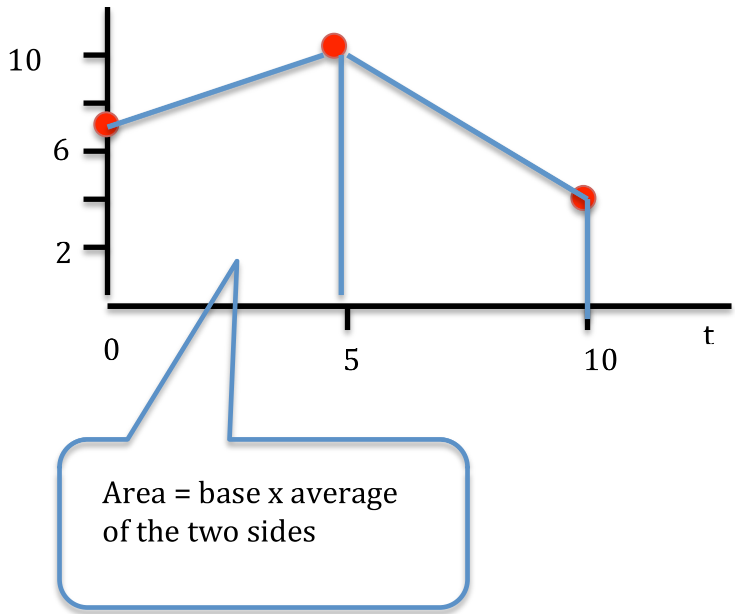
Complete approximation of flow:

$$5\,\frac{7 + 10}{2} + 5\,\frac{10 + 4}{2} = 77.3\ m^3$$

This is called the “Trapezoidal Method”.

It gives better approximations as we shorten the intervals for measurement.

Where do the trapezoids come in?




So

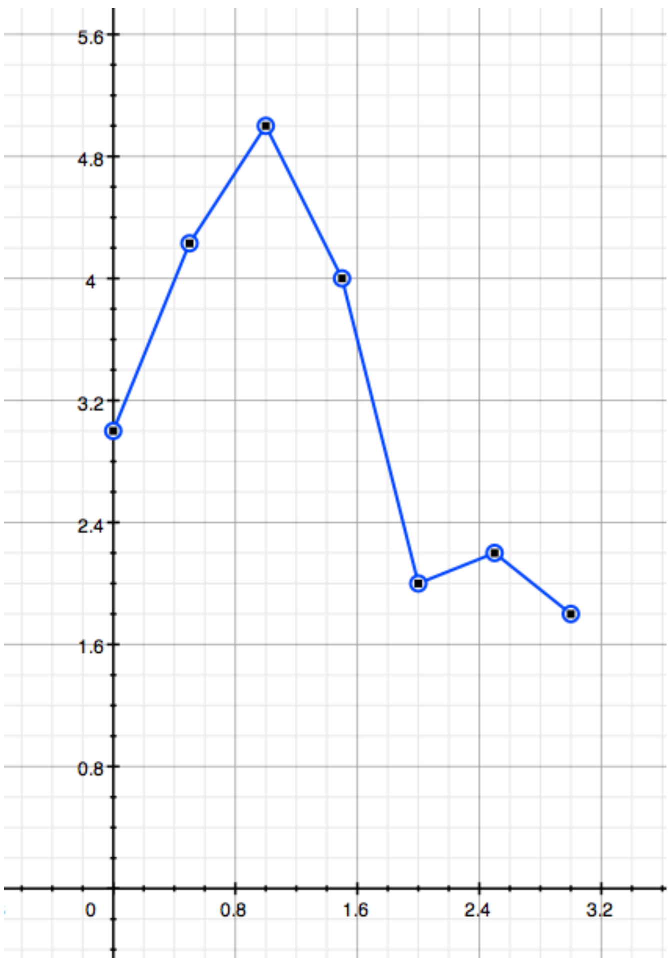
$$5\frac{7+10}{2}+5\frac{10+4}{2}=77.3\ m^3$$

is the sum of the areas of the two trapezoids.

**Example 2:**

Measurement interval = 0.5

| | |
|---|---|
| 0 | 3 |
| 0.5 | 4.23 |
| 1 | 5 |
| 1.5 | 4 |
| 2 | 2 |
| 2.5 | 2.2 |
| 3 | 1.8 |




| | |
|---|---|
| 0 | 3 |
| 0.5 | 4.23 |
| 1 | 5 |
| 1.5 | 4 |
| 2 | 2 |
| 2.5 | 2.2 |
| 3 | 1.8 |

$$0.5\times\left(\frac{3+4.23}{2}+\frac{4.23+5}{2}+\frac{5+4}{2}+\frac{4+2}{2}+\frac{2+2.2}{2}+\frac{2.2+1.8}{2}\right)=9.92$$


Suppose the black curve is the actual flow rate.

How well did we do?

**Calculation**

$$0.5\times\left(\frac{3+4.23}{2}+\frac{4.23+5}{2}+\frac{5+4}{2}+\frac{4+2}{2}+\frac{2+2.2}{2}+\frac{2.2+1.8}{2}\right)$$

$$0.5\times\left(\frac{3}{2}+4.23+5+4+2+2.2+\frac{1.8}{2}\right)$$

**Summary**

1. The Trapezoidal Method gives an approximate value for the integral = the total flow

2. The Trapezoidal Method uses a straight-line approximation of the curve.

3. This is a numerical technique; it gives a number

4. Using more and shorter measurement intervals gives better estimates

5. Many modeling situations can only be analyzed by numerical methods